Lecture 6

Nonlinear Univariate Characteristics

Digital Signal Processing and Analysis in Biomedical Systems

Contents

Signal Analysis approaches

- Linear / non-linear
- Univariate / multivariate

	Linear	Non-linear
Univariate	$R = F[s(t)]$, $F[as_1(t) + bs_2(t)] = aF[s_1(t)] + bF[s_2(t)]$	$R = F[s(t)]$, $F[as_1(t) + bs_2(t)] \neq aF[s_1(t)] + bF[s_2(t)]$
Multivariate	$R = F[s_1(t), s_2(t), s_3(t), \ldots, s_N(t)]$, $F[as_1(t) + bs_2(t)] = aF[s_1(t)] + bF[s_2(t)]$	$R = F[s_1(t), s_2(t), s_3(t), \ldots, s_N(t)]$, $F[as_1(t) + bs_2(t)] \neq aF[s_1(t)] + bF[s_2(t)]$

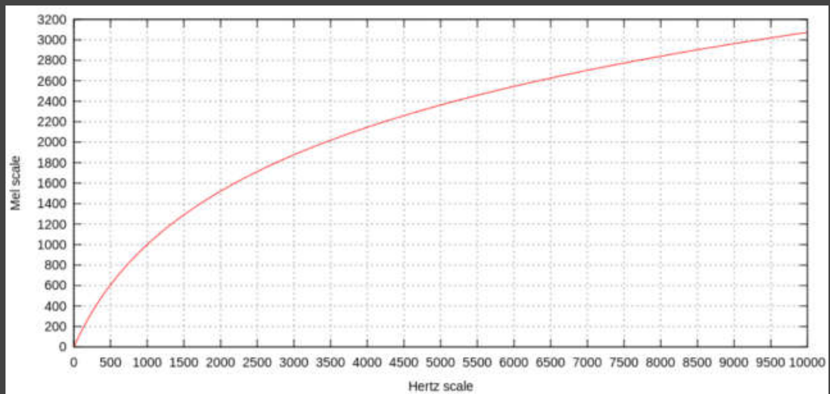
Non-linear Univariate analysis

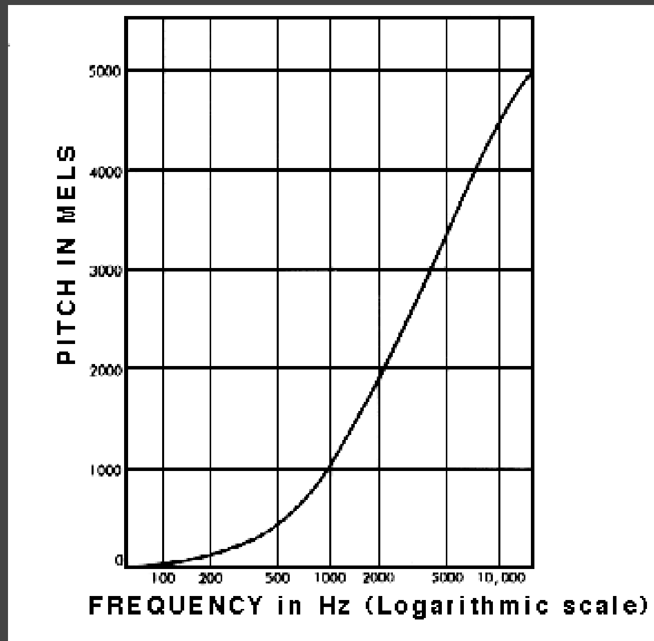
The superposition principle does not hold for the nonlinear analysis:

- The analysis results are not proportional to the input signals.
- The results for sum of the signals are not the sum of initial parameters.

Human hearing and speech recognition using cepstral analysis

Human hearing is linear only to 1000 Hz, and non-linear (logarithmic) on higher frequencies.





Application to speech recognition

the speech signal can be modeled as the convolution of glottal source $u[n]$, vocal tract $v[n]$, and radiation $r[n]$

$$y[n] = u[n] \otimes v[n] \otimes r[n]$$

- Because these signals are convolved, they cannot be easily separated in the time domain

We can however perform the separation as follows

- For convenience, we combine $v'[n] = v[n] \otimes r[n]$, which leads to
$$y[n] = u[n] \otimes v'[n]$$
- Taking the Fourier transform
$$Y(e^{j\omega}) = U(e^{j\omega})V'(e^{j\omega})$$
- If we now take the log of the magnitude, we obtain
$$\log(|Y(e^{j\omega})|) = \log(|U(e^{j\omega})|) + \log(|V'(e^{j\omega})|)$$
  - which shows that source and filter are now just added together
- We can now return to the time domain through the inverse FT
$$c[n] = c_u[n] + c_v[n]$$

Cepstral analysis – 2

$$x[n] = x_1[n] * x_2[n] \Longleftrightarrow X(z) = X_1(z)X_2(z)$$

↓ *Complex logarithm*

$$\log\{X(z)\} = \log\{X_1(z)\} + \log\{X_2(z)\} = \hat{X}(z)$$

↓ *Inverse Z transform*

$$\hat{x}(n) = \hat{x}_1(n) + \hat{x}_2(n)$$

↓ *In unit circle |z|<1*

$$\hat{X}(e^{j\omega}) = \log|X(e^{j\omega})| + j \ \arg\{X(e^{j\omega})\}$$

Cepstrum

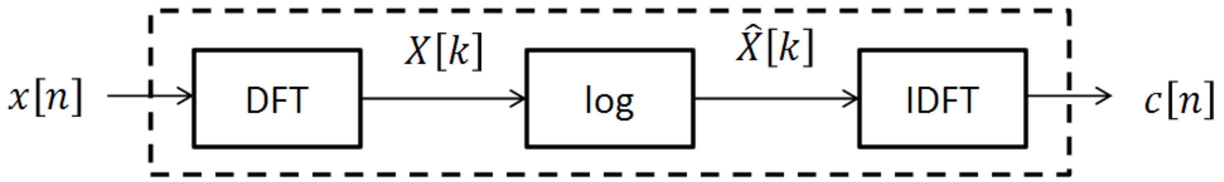
– The cepstrum is defined as the inverse DFT of the log magnitude of the DFT of a signal

$$c[n] = \mathcal{F}^{-1}\{\log|\mathcal{F}\{x[n]\}|\}$$

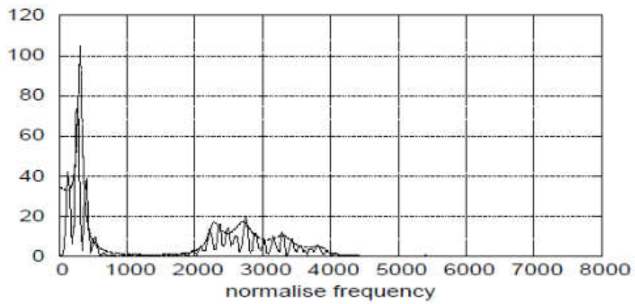
- where $\mathcal{F}$ is the DFT and $\mathcal{F}^{-1}$ is the IDFT

– For a windowed frame of speech $y[n]$, the cepstrum is

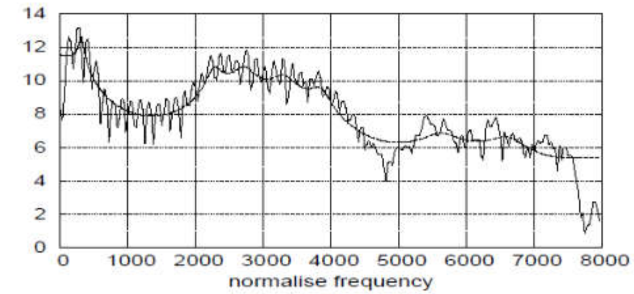
$$c[n] = \sum_{n=0}^{N-1} \log\left(\left|\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}\right|\right) e^{j\frac{2\pi}{N}kn}$$



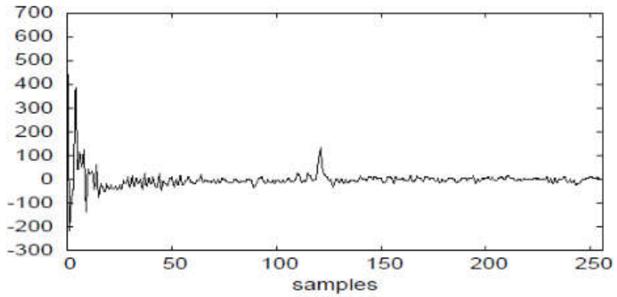
$\mathcal{F}\{x[n]\}$



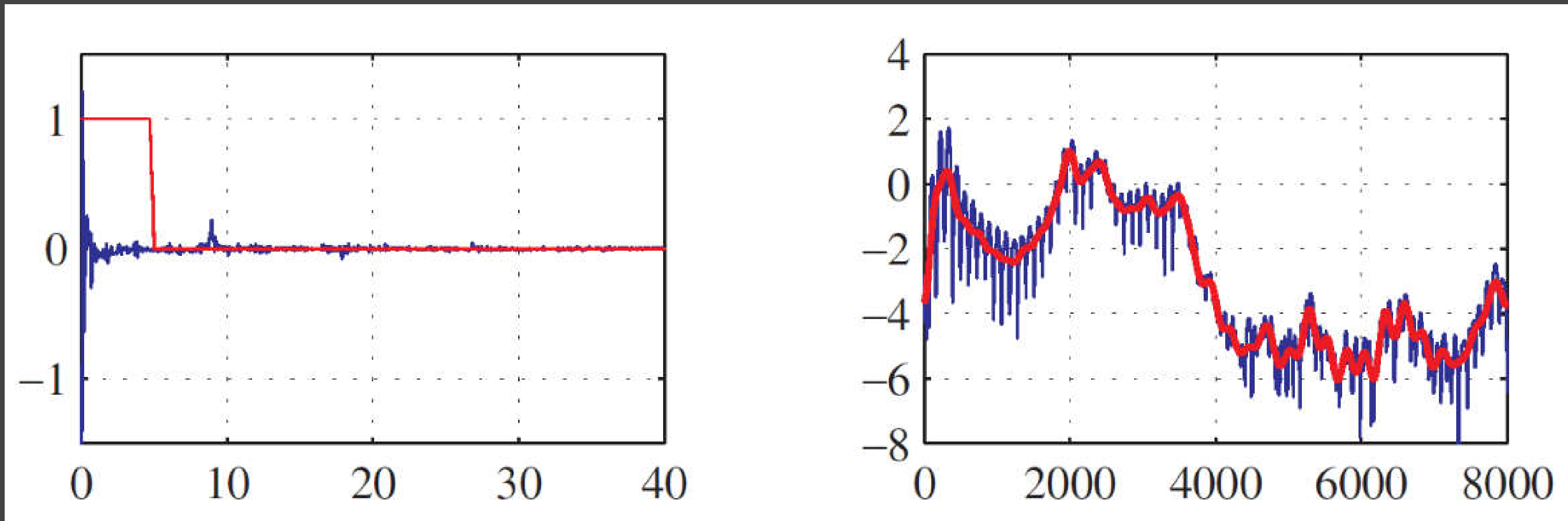
$\log|\mathcal{F}\{x[n]\}|$



$\mathcal{F}^{-1}\{\log|\mathcal{F}\{x[n]\}|\}$



LIFTering in CEPSTral domain



Notices (cepstral analysis)

- the log spectrum can be treated as a waveform and subjected to further Fourier analysis
- independent variable of the cepstrum is nominally time since it is the IDFT of a log-spectrum, but is interpreted as a frequency since we are treating the log spectrum as a waveform
- to emphasize this interchanging of domains, Bogert, Healy and Tukey (1960) coined the term cepstrum by swapping the order of the letters in the word Spectrum
– the name of the independent variable of the cepstrum is known as a quefrency, and the linear filtering operation in the previous slide is known as liftering

Autocorrelation analysis

The aim of autocorrelation is to unveil the degree of self-similarity or cyclic behavior of the process. In ideal case, the periodic signal is perfectly autocorrelated, so the period can be defined.

Autocorrelation is the correlation of a signal with a delayed copy of itself as a function of delay.

Usually autocrrelation is used in case of presence of noise, when the cyclic behavior is not obvious.

Autocorrelation function

Given measurements, Y_1, Y_2, ..., Y_N at time X_1, X_2, ..., X_N, the lag *k* autocorrelation function is defined as

$$r_k = \frac{\sum_{i=1}^{N-k}(Y_i - \bar{Y})(Y_{i+k} - \bar{Y})}{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}$$

Wiener-Khinchin Theorem – 1

Let x(t) be a real wide-sense stationary process with autocorrelation function

$$R_x(\tau) = Ex(t)x(t+\tau)$$

and assume that it satisfies the Dirichlet conditions (it is absolutely integrable, this integral converges:

$$\int_{-\infty}^{\infty} |R_x(\tau)| d\tau$$

and its Fourier Transform exists:

$$\int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

Wiener-Khinchin Theorem – 2

Now for each sample function x(t) the truncated Fourier transform can be defined:

$$X_T(f) \triangleq \int_{-T/2}^{T/2} x(t) e^{-j2\pi f t} dt$$

The corresponding power spectral density is $\frac{1}{T} \left| X_T(f) \right|^2$

Its expectation is

$$S_T(f) \triangleq E \frac{1}{T} \left| X_T(f) \right|^2$$

Wiener-Khinchin Theorem – 3

power spectral density of the process x(t) is therefore:

$$S_x(f) = \lim_{T \to \infty} S_T(f)$$

Theorem: For all *f*, the above limit exists and

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f \tau} d\tau$$

Wiener-Khinchin Theorem – 4

Discrete time case:

$$S(f) = \sum_{k=-\infty}^{\infty} r_{xx}[k] e^{-i(2\pi f)k},$$

where

$$r_{xx}[k] = \mathrm{E}\left[\, x[n] x^*[n-k] \,\right]$$

Wiener-Khinchin Theorem – 5, applications

Used when the inputs and outputs of Linear Time-invariant System are not square integrable, so their Fourier transforms do not exist.

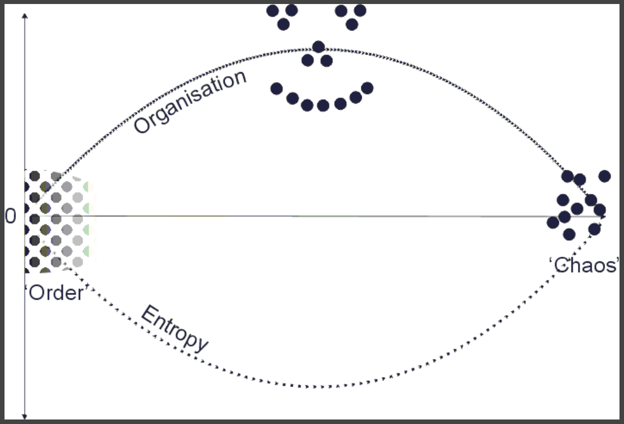
The power spectrum of the output is equal to the power spectrum of the input times the energy transfer function of the system.

Entropy analysis

Entropies are among the most popular and promising complexity measures for biological signal analyses.

In physics, the entropy is the measure of the disorder and uncertainty in the system.

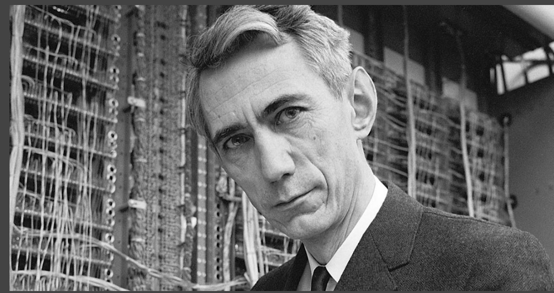
Entropy analysis of the signal gives the degree of unpredictedness, chaoticity, complexity, randomness, disorder of time series.



Types of entropy

- Shannon entropy
- Permutation Entropy (PE),
- Approximate Entropy (ApEn),
- Fuzzy Entropy (FE),
- Sample Entropy (SampEn),
- Renyi's Entropy (RE),
- Spectral Entropy (SEN),
- Wavelet Entropy (WE),
- Tsallis entropy (TE),
- Higher Order Spectra Entropies
- (S1, S2, PhEn),
- Kolmogorov–Sinai entropy (KSE)
- Recurrence Quantification Analysis entropy (RQA En)

Shannon entropy



Proposed by Claude Shannon in 1948, in the paper “The mathematical theory of communication” (Bell System Technical Journal):

Shannon entropy *H* is given by the formula

$$H = -\sum_{i} p_i \log_b p_i$$

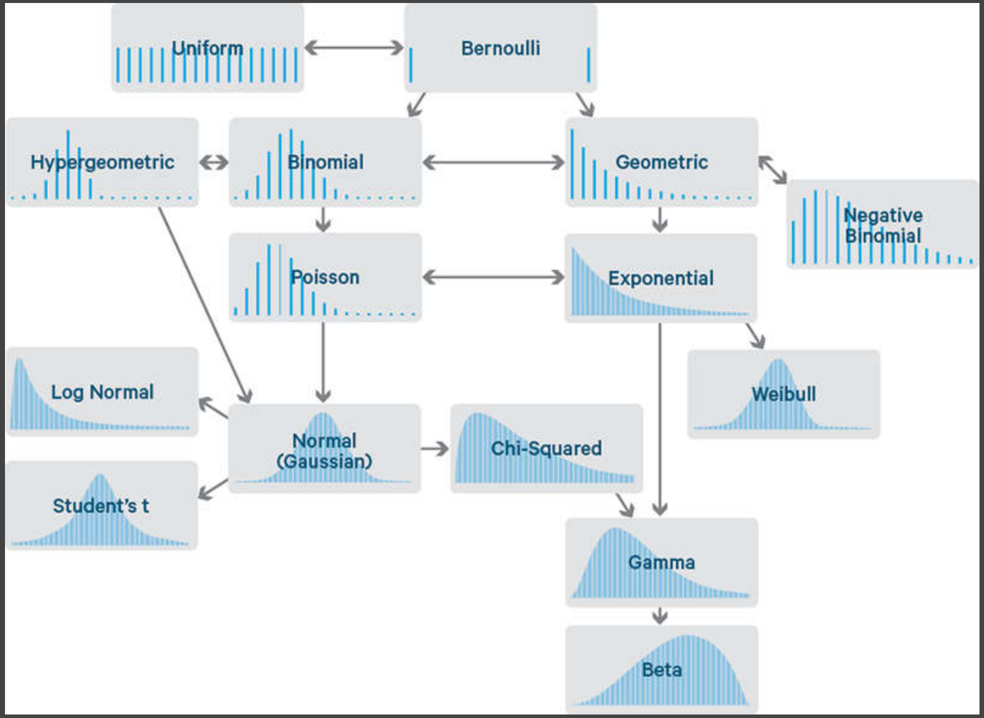
where p_i is the probability of character number *i* showing up in a stream of characters of the given "script".

Entropy for digital signals

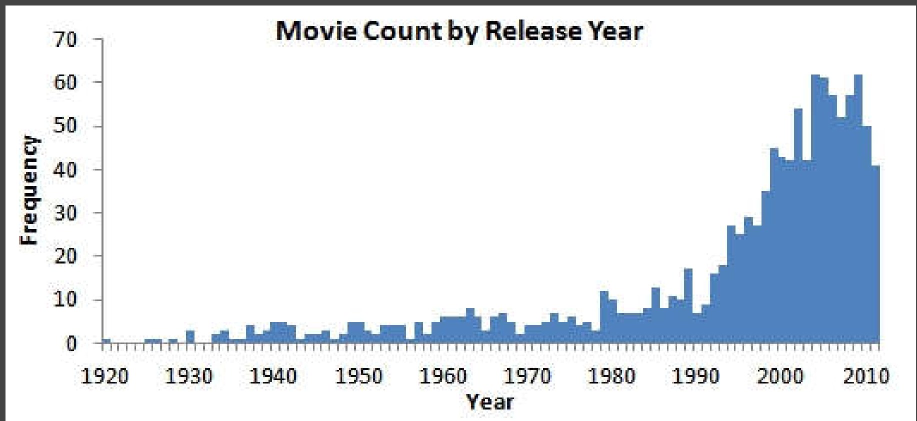
The signal values are considered as the random sequence.

Probability Density Function

$$F(x) = P(a \le x \le b) = \int_a^b f(x)dx \ge 0$$



Histograms



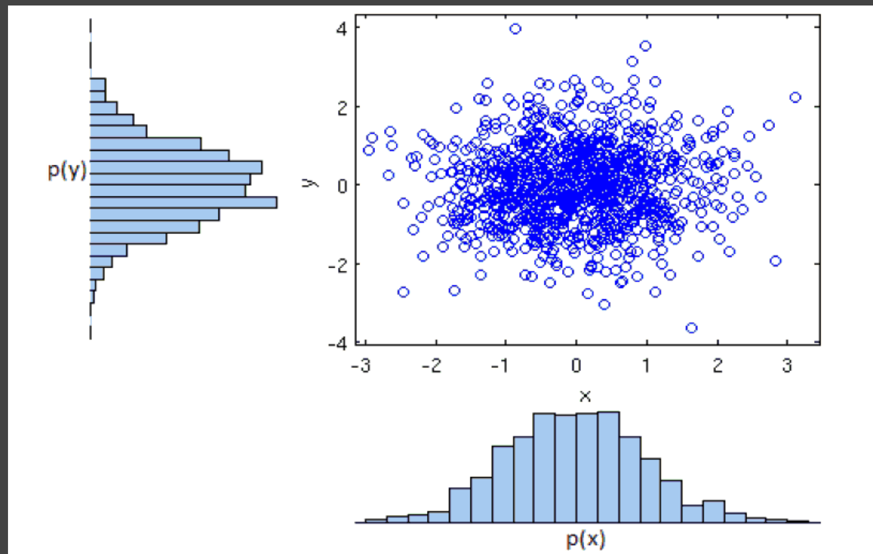


Fig. 1. Marginal density functions of Gaussian distributed random variables X and Y and observations from the joint probability distribution (top, sample size $N = 10^3$) are shown

Bin Number:

$$k = \left\lceil \frac{\max x - \min x}{h} \right\rceil.$$

Selection of bin size – 1

Sturge's Rule is the most commonly used (implicitly assumes approximately normal distribution):

$$k = 1 + \log_2 n$$

Scott's normal reference rule (optimal for random samples from normally distributed data):

$$h = \frac{3.5\sigma}{n^{1/3}}$$

Selection of bin size – 2

Freedman-Diaconis rule (based on interquartile range):

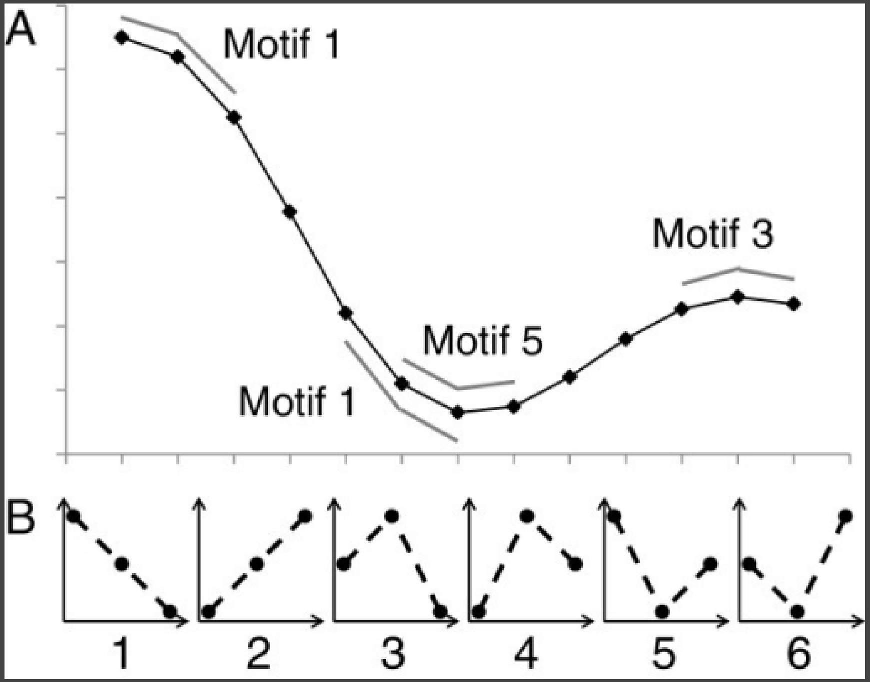
$$h = 2\frac{\mathrm{IQR}(x)}{n^{1/3}}$$

Shimazaki rule (based on minimization of an estimated risk function):

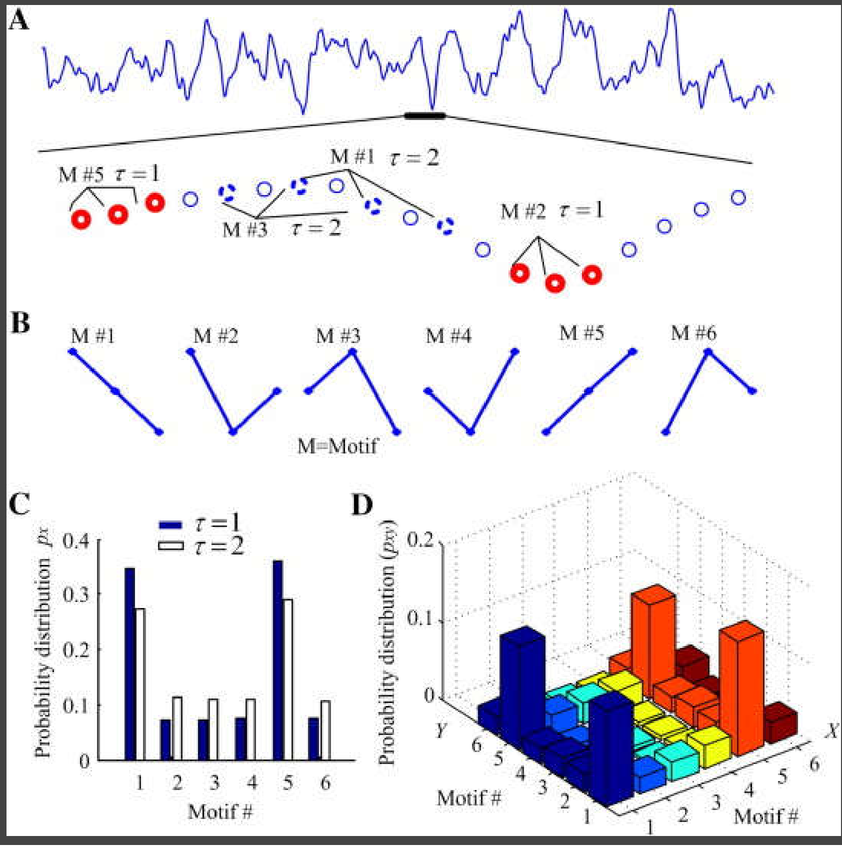
$$\arg\min_h \frac{2\bar{m} - v}{h^2}$$

Permutation Entropy – 1

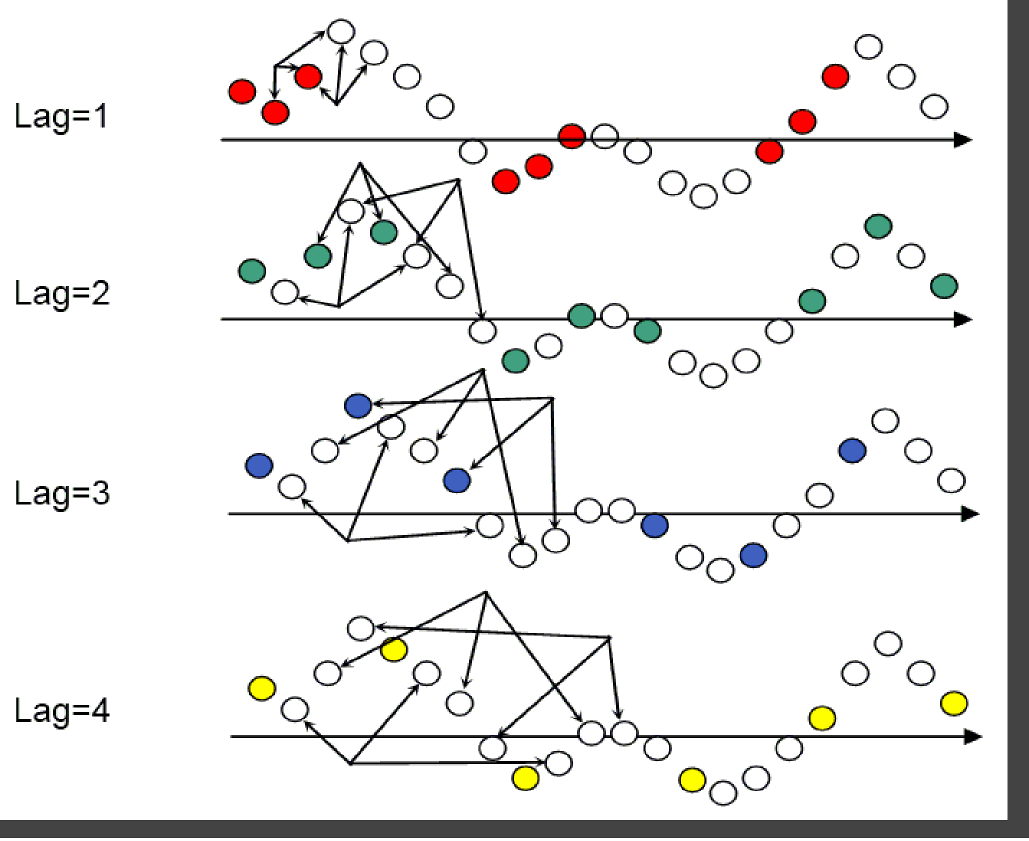
PE is a descriptor of time series *complexity, unpredictability, disorder, chaoticity, nonlinearity and stochasticity* and can reflect complex nonlinear interconnections between anatomical and functional subsystems emerged in brain and other systems in healthy state and during various diseases.



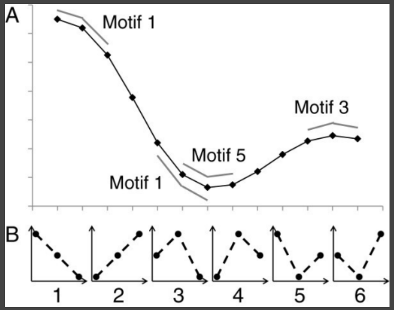
Permutation Entropy – 2



Permutation Entropy – 3



Permutation entropy – 4



Relative number of motifs:

$$p(\pi_j) = \frac{q(j)}{N-(m-1)l}$$

Maximum value of is reached for a uniform distribution on all permutations:

$$PE(m) = \ln(m!) \text{ when } p_k = {}^{1}/_{m!} \text{ for all } k = 1, 2, \ldots, m!$$

Permutation Entropy, normalized:

$$PermEn_{norm}(m,l) = \frac{-\sum_{j=1}^{m!} p(\pi_j) \log p(\pi_j)}{\log m!}$$

Modified Permutation Entropy – 1

Modified PE is well defined also in the case of tied ranks. That means, some values of the motifs might be equal.

Thus get more than m! ordered patterns.

For example, if m=3 we get m!=6 order patterns (1; 2; 3), (1; 3; 2), (2; 1; 3), (2; 3; 1), (3; 1; 2), (3; 2; 1) representing all vectors with no ties,

and 7 additional patterns for the case of ties, (1; 2; 2), (2; 1; 1), (1; 2; 1), (2; 1; 2), (1; 1; 2), (2; 2; 1), and (1; 1; 1).

Modified Permutation Entropy – 2

In the general case, the number of possible order patterns is calculated by the Bell number:

$$B(m) = \sum_{r=0}^{m}\left(\sum_{s=0}^{r}(-1)^{r-s}\frac{r!}{s!\,(r-s)!}s^{m}\right)$$

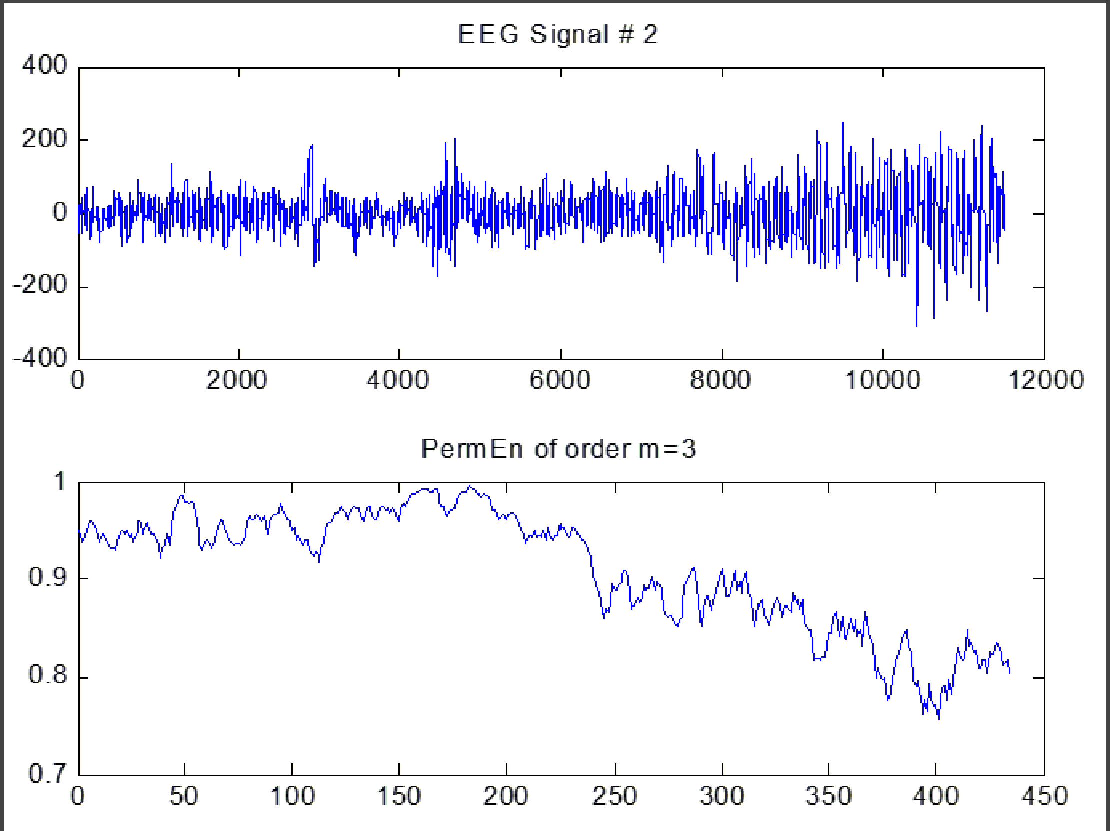
Relationship between embedding dimension and Bell number

m	2	3	4	5	6	7
$m!$	2	6	24	120	720	5,040
$B(m)$	3	13	75	541	4,683	47,293
$B(m)/m!$	1.5	2.16	3.125	4.50	6.50	9.38

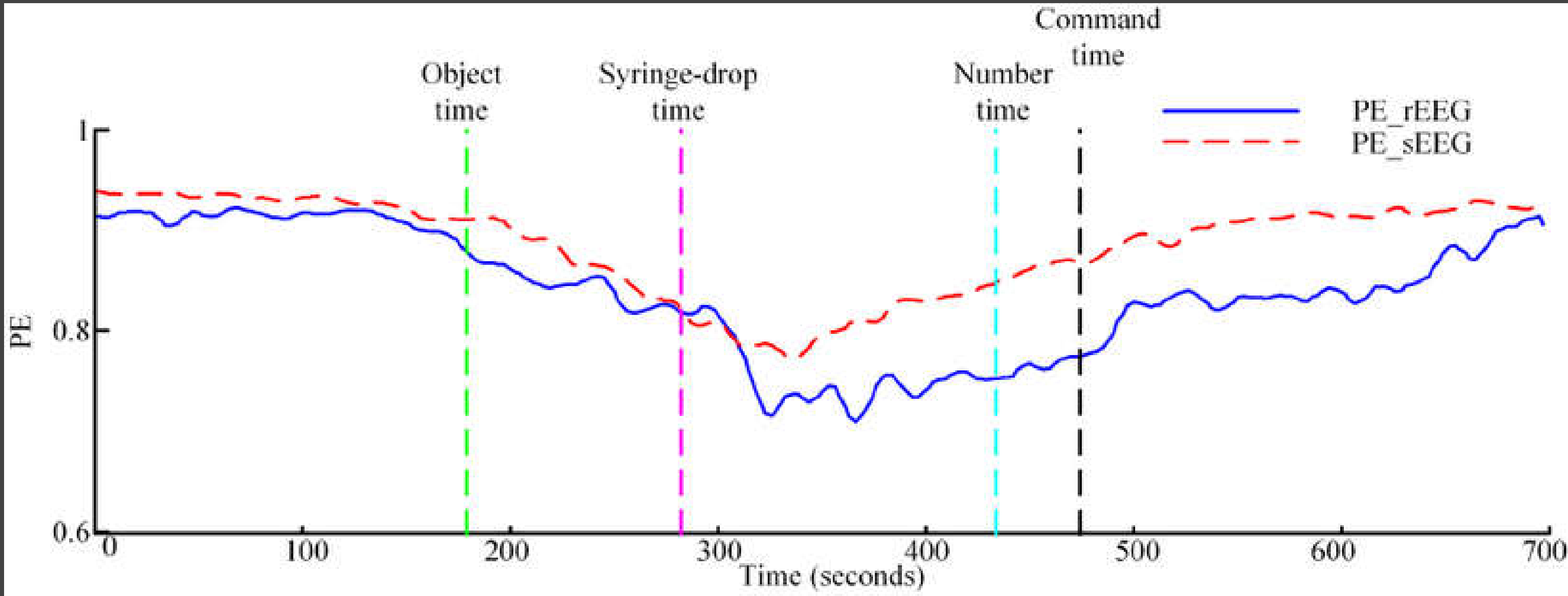
Normalized Modified PE:

$$nmPE\ (m) = \frac{mPE\ (m)}{B\ (m)}$$

Permutation Entropy for seizure prediction



Permutation Entropy for anaesthesia



Chaotic characteristics

DFA

Detrended Fluctuation Analysis

Respiration patterns